Exercise 17D: Solutions

1 **a** $|\bmit a| = \sqrt{1+9} = \sqrt{10}$

$\therefore \hat{\bmit a} = \frac{1}{\sqrt{10}}(\bmit i + 3\bmit j)$

b $|\bmit b| = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

$\therefore \hat{\bmit a} = \frac{1}{2\sqrt{2}}(2\bmit i + 2\bmit j) = \frac{1}{\sqrt{2}}(\bmit i + \bmit j)$

c $\bmit c = \overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB} = \bmit i - \bmit j$

$\therefore \hat{\bmit c} = \frac{1}{\sqrt{2}}(\bmit i - \bmit j)$

2 **a** **i** $\hat{\bmit a} = \frac{1}{5}(3\bmit i + 4\bmit j)$

ii $|\bmit b| = \sqrt{2}$

b $\frac{\sqrt{2}}{5}(3\bmit i + 4\bmit j)$

3 **a** **i** $\hat{\bmit a} = \frac{1}{5}(3\bmit i + 4\bmit j)$

ii $\hat{\bmit b} = \frac{1}{13}(5\bmit i + 12\bmit j)$

b Let $\overrightarrow{OA'} = \hat{\bmit a}$ and $\overrightarrow{OB'} = \hat{\bmit b}$

Then $\triangle A'OB'$ is isosceles. Therefore the angle bisector of $\angle AOB$ passes through the midpoint of $A'B'$.

Let M be the midpoint of $A'B'$

Then

$$\begin{aligned}\overrightarrow{OM} &= \frac{1}{2}(\hat{\bmit a} + \hat{\bmit b})\\ &= \frac{1}{2}(\frac{1}{5}(3\bmit i + 4\bmit j) + \frac{1}{13}(5\bmit i + 12\bmit j))\\ &= \frac{8}{65}(4\bmit i + 7\bmit j)\end{aligned}$$

$\therefore$the unit vector in the direction of

$\overrightarrow{OM}$ is: $= \frac{1}{\sqrt{65}}(4\bmit i + 7\bmit j)$

4 **a** $\bmit a = \bmit i + 3\bmit j, \bmit b = \bmit i - 4\bmit j$

$$\begin{aligned}\frac{\bmit a \cdot \bmit b}{\bmit b \cdot \bmit b}\bmit b &= \frac{1-12}{17}(\bmit i - 4\bmit j)\\ &= -\frac{11}{17}(\bmit i - 4\bmit j)\end{aligned}$$

b $\bmit a = \bmit i - 3\bmit j, \bmit b = \bmit i - 4\bmit j$

$$\begin{aligned}\frac{\bmit a \cdot \bmit b}{\bmit b \cdot \bmit b}\bmit b &= \frac{1+12}{17}(\bmit i - 4\bmit j)\\ &= \frac{13}{17}(\bmit i - 4\bmit j)\end{aligned}$$

c The vector resolute is $\bmit b$

5 **a** $\frac{\bmit a \cdot \bmit b}{|\bmit b|} = \frac{2}{1} = 2$

b $\frac{\bmit a \cdot \bmit c}{|\bmit c|} = \frac{3-2}{\sqrt{5}} = \frac{1}{\sqrt{5}}$

c $\frac{\bmit a \cdot \bmit b}{|\bmit a|} = \frac{2\sqrt{3}}{\sqrt{7}}$

d $\frac{\bmit b \cdot \bmit c}{|\bmit c|} = \frac{-1-4\sqrt{5}}{\sqrt{17}}$

6 **a** $\boldsymbol{a} = \boldsymbol{u} + \boldsymbol{w}$ where $\boldsymbol{u} = 2\boldsymbol{i}$ and $\boldsymbol{w} = \boldsymbol{j}$

b $\boldsymbol{a} = \boldsymbol{u} + \boldsymbol{w}$ where $\boldsymbol{u} = 2\boldsymbol{i} + 2\boldsymbol{j}$ and $\boldsymbol{w} = \boldsymbol{i} - \boldsymbol{j}$

c $\boldsymbol{a} = \boldsymbol{u} + \boldsymbol{w}$ where $\boldsymbol{u} = \boldsymbol{0}$ and $\boldsymbol{w} = -\boldsymbol{i} + \boldsymbol{j}$

7 **a** $\frac{\bmit a \cdot \bmit b}{\bmit b \cdot \bmit b} \bmit b = 2(\bmit i + \bmit j)$

b Let $\overrightarrow{OC} = 2(\bmit i + \bmit j)$

$\overrightarrow{OC}$ is the vector resolute of $\bmit a$ in the direction of $\bmit b$

$\therefore \overrightarrow{CA}$ is a vector perpendiculr to $\overrightarrow{OB}$

$$\begin{aligned}\overrightarrow{CA} &= \overrightarrow{CO} + \overrightarrow{OA} \\ &= -2(\bmit i + \bmit j) + (\bmit i + 3\bmit j) \\ &= -\bmit i + \bmit j\end{aligned}$$

Therefore the unit vector is $\frac{1}{\sqrt{2}}(-\bmit i + \bmit j)$

8 **a** $\frac{\bmit a \cdot \bmit b}{\bmit b \cdot \bmit b} \bmit b = \frac{3}{2}(\bmit i - \bmit j)$

b $$\begin{aligned}\bmit a - \frac{\bmit a \cdot \bmit b}{\bmit b \cdot \bmit b} \bmit b &= 4\bmit i + \bmit j - \frac{3}{2}(\bmit i - \bmit j) \\ &= \frac{1}{2}(8\bmit i + 2\bmit j - 3\bmit i + 3\bmit j) \\ &= \frac{1}{2}(5\bmit i + 5\bmit j)\end{aligned}$$

c Distance = $|\frac{1}{2}(5\bmit i + 5\bmit j)| = \frac{5\sqrt{2}}{2}$

9 $\overrightarrow{OA} = \bmit a = \bmit i + 2\bmit j$

$\overrightarrow{OB} = \bmit b = 2\bmit i + \bmit j$

$\overrightarrow{OC} = \bmit c = 2\bmit i - 3\bmit j$

a **i** $$\begin{aligned}\overrightarrow{AB} &= \overrightarrow{AO} + \overrightarrow{OB} \\ &= -\bmit i - 2\bmit j + 2\bmit i + \bmit j \\ &= \bmit i - \bmit j\end{aligned}$$

ii $$\begin{aligned}\overrightarrow{AC} &= \overrightarrow{AO} + \overrightarrow{OC} \\ &= -\bmit i - 2\bmit j + 2\bmit i - 3\bmit j \\ &= \bmit i - 5\bmit j\end{aligned}$$

b $$\begin{aligned}\text{The vector resolute } &= \frac{\overrightarrow{AB} \cdot \overrightarrow{AC}}{\overrightarrow{AC} \cdot \overrightarrow{AC}} \overrightarrow{AC} \\ &= \frac{1+5}{26}(\bmit i - 5\bmit j)\end{aligned}$$

$$= \frac{3}{13}(\backslash\mathrm{bmit}i - 5\backslash\mathrm{bmit}j)$$

c $\text{The shortest distance} = \overrightarrow{AB} - \frac{3}{13}(\backslash\mathrm{bmit}i - 5\backslash\mathrm{bmit}j)$

$$= \frac{3}{13}(10\backslash\mathrm{bmit}i + 2\backslash\mathrm{bmit}j)$$

The shortest distance is the height of triangle ABC wherethe base is taken as AC

Therefore height $= |\frac{3}{13}(10\backslash\mathrm{bmit}i + 2\backslash\mathrm{bmit}j)| = \frac{1}{13}\sqrt{104} = \frac{2}{13}\sqrt{26}$

d The area of the triangle

$= \frac{1}{2} \times \frac{1}{13}\sqrt{104} \times \sqrt{26}$

=2